Sec 2 عمليات عشوائية

Distributive Law :-

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

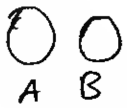
Demorgan's Law :-

$(A \cup B)^c = A^c \cap B^c$

$(A \cap B)^c = A^c \cup B^c$

Theorem 3

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$



$A \cap B = \phi \rightarrow$ mutually exclusive

$P(\phi) = 0$

Ex: Prove that $P(A^c) = 1 - P(A)$

$P(A \cup A^c) = 1$

$P(A) + P(A^c) - \underbrace{P(A \cap A^c)}_{\rightarrow 0} = 1$

$P(A) + P(A^c) = 1$

$P(A^c) = 1 - P(A)$

① A, B Express and Venn-diagram

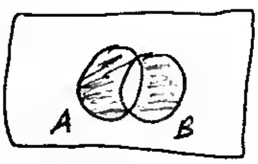
a) A but not B

$A - B$

b) Either A or B but ~~both~~ not both

$\rightarrow (A \cup B) - (A \cap B)$

$\rightarrow (A - B) \cup (B - A)$

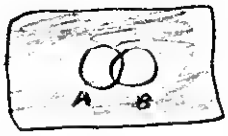


c) A or not B

$A \cup B^c$

d) Neither A nor B

$(A \cup B)^c$



② A, B, C

a) A and B but not C

$(A \cap B) - C$

b) only A occurs

$\rightarrow A - (B \cup C)$

$\rightarrow (A - B) \cap (A - C)$

$\rightarrow (A - B) - C$

③ Coin and die

a) Express

A: head and even

$A = \{(H,2), (H,4), (H,6)\}$

B: prime number

$B = \{(H,2), (H,3), (H,5), (T,2), (T,3), (T,$

C: tail and odd

$C = \{(T,1), (T,3), (T,5)\}$

b) Express

i) A or B

$A \cup B$

ii) B and C

$B \cap C = \{(T,3)(T,5)\}$

iii) only B

$B - A - C = \{(H,3), (H,5), (T,2)\}$

c) Mutually Exclusive events

A , C

(4) $P(H) = P(T) * 2$

find $P(H)$, $P(T)$

$$\boxed{\begin{aligned} P(H) &= \frac{2}{3} \\ P(T) &= \frac{1}{3} \end{aligned}}$$

$P(H) + P(T) = 1$

$2P(T) + P(T) = 1$

$3P(T) = 1$

$P(T) = \frac{1}{3}$

$P(H) = 1 - \frac{1}{3}$

$P(H) = \frac{2}{3}$

(7) 10 boys and 20 girls

half of boy and girls have brown eyes

P(boy or has brown eyes) = ?

E_1: boys $\rightarrow P(E_1) = \frac{10}{30} = \frac{1}{3}$

E_2: brown eyes $\rightarrow P(E_2) = \frac{1}{2}$

$P(E_1 \cap E_2) = \frac{5}{30} = \frac{1}{6}$

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{1}{3} + \frac{1}{2} - \frac{1}{6}$

$= \frac{2}{3}$

Theorem:- sample point $= (c_1, c_2, \cdots)$

P(sample point) $= P(c_1) * P(c_2) * \ldots$

(10) Find the probability of an event if the odds that it will occur are a : b that is; a to b

$P(\text{Event}) = \frac{a}{a+b}$

(11) 3 to 2

$P(\text{Event}) = \frac{3}{5}$

(8) die is weighted so that the probability of ~~number~~ a number appearing when the die is tossed is proportional to the given number

A : even no.

B : prime no.

C : odd no

a) find the probability of each sample point

b) P(A), P(B), P(C)

c) ----

(a)

$P(1) = \frac{1}{21}$ $P(2) = \frac{2}{21}$

$P(3) = \frac{3}{21}$ $P(4) = \frac{4}{21}$

$P(5) = \frac{5}{21}$ $P(6) = \frac{6}{21}$

(b)

(19) Prove that

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$
$- P(A \cap B) - P(A \cap C) - P(A \cap C)$
$+ P(A \cap B \cap C)$

$P(A \cup (B \cup C)) = P(A) + P(B \cup C) - P(A \cap (B \cup C))$

$= P(A) + P(B) + P(C) - P(B \cap C)$
$- P((A \cap B) \cup (A \cap B))$

$= P(A) + P(B) + P(C) - P(B \cap C)$
$- P(A \cap B) - P(A \cap C) + \underbrace{P((A \cap B) \cap (A \cap C))}_{P(A \cap B \cap C)}$

Report $\rightarrow$ 5, 6, 8, 12, 13, 15, 20